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ANTENNA DESIGN AS A PARTIAL BASIS PROBLEM.(U)

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(6) Antenna Design as a Partial Basis Problem*

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Antenna Design as a Partial Basis Problem

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1. INTRODUCTION

In this paper we discuss an algorithm for choosing a best "partial basis" $h_{i_1},\cdots,h_{i_n}$ from a collection of basis functions $h_1,\cdots,h_n$ to best approximate a given function f with respect to a specified norm. Our results have been strongly motivated by the difficult problem of choosing best locations for antenna elements in a linear array. This problem, which has been considered in various forms in the engineering literature ([1]-[4]), is formulated below so that the reader can better understand the subsequent developments. In section 2 we formulate the partial basis problem precisely, present an algorithm for its solution, and discuss numerical results of the application of the algorithm to the antenna design problem. In section 3 some theoretical results are presented.

The field pattern of a symmetric line array of antenna elements with real symmetric element currents is proportional to the magnitude of

$$(1) \qquad p(u) = \sum_{k=1}^{n} a_k \cos \xi_k u \ , \quad 0 \le u \le \pi \ ,$$

where $u = \pi \sin\theta$, θ is the angle measured from a normal to the array axis, $\xi_k = x_k/(\lambda/2)$, λ is the wavelength of the design frequency, and x_k is the distance of the kth element from the center of the array. If the array aperture is constrained to be at most A, say, then we will have $0 \le x_1 < \cdots < x_n \le A/2$. The basic objective is to vary the element currents a_k and locations ξ_k to make $|p(u)|$ small for u away from $u=0$, subject to the normalization condition $|p(0)| = 1$. We formulate this as a problem of approximating an ideal function

$$(2) \qquad f(u) = \begin{cases} 1 \text{ if } 0 \le u \le u_0 \\ 0 \text{ if } u_1 < u \le \pi \end{cases}$$

where $0 < u_0 \le u_1 < \pi$ (if $u_0 < u_1$, we define $f(u_1) = 0$ also). As a measure of the approximation we use an L_r norm, $1 \le r < \infty$,

$$(3) \qquad || f - p || = \left\{ \int_{[0,u_0] \cup [u_1,\pi]} w(u) |f(u) - p(u)|^r du \right\}^{1/r},$$

the most important case being $r = 2$ (least squares approximation), or a uniform (minimax) norm

$$(4) \qquad ||f - p|| = \max_{u \epsilon [o,u_0] \cup [u_1,\pi]} w(u) \mid f(u) - p(u) \mid .$$

Here $w(u)$ is a continuous weight function, often chosen to be $w(u) \equiv 1$.

Our formulation of the antenna design problem is then

$$(5)\quad \begin{cases} \underset{a_k,\ \xi_k}{\text{minimize}} & \| f(u) - \sum_{k=1}^{n} a_k \cos \xi_k u \| \\ \text{subject to} & 0 \le \xi_1 < \cdots < \xi_n \le L \\ \text{and} & \xi_{k+1} - \xi_k \ge \Delta > 0,\ k = 1,\cdots,n-1 \end{cases}$$

where $\Delta > 0$ is specified and L serves to limit the aperture of the array. The condition $\xi_k - \xi_{k-1} \ge \Delta$ prevents any two elements from getting arbitrarily close together. This assumption is natural on physical grounds; furthermore it has an important effect in simplifying the mathematics.

An important feature of our approach to problem (5) is restricting the variables $\xi_1,\cdots,\xi_n$ to a discrete set $T = \{t_1,\cdots,t_N\}$ of possible values, where $0 = t_1<\cdots<t_N = L$, $N > n$, and $t_{i+1} - t_i \ge \Delta$, $i = 1,\cdots,N-1$. The most obvious choice for T would be equispaced points $t_i = (i-1)L/(N-1)$, $i = 1,\cdots,N$. The problem (5) now becomes

$$(6)\qquad \underset{\substack{a_1,\cdots,a_n \\ \{\xi_1,\cdots,\xi_n\} \subset T}}{\text{minimize}} \quad \| f(u) - \sum_{k=1}^{n} a_k \cos \xi_k u \| .$$

Since each t_i gives rise to a basis function $\cos t_i u$, we think of problem (6) as that of choosing a best partial basis $\{\cos \xi_i u : i = 1,\cdots,n\}$ from the full set of possible basis functions $\{\cos t_i u : i = 1,\cdots,N\}$ to approximate the ideal function f with respect to the specified norm.

2. THE ALGORITHM FOR THE PARTIAL BASIS PROBLEM

We now extend the partial basis problem (6) for the selection of best antenna element locations and currents to a more general context. Let $h_1, \cdots, h_N$ be linearly independent basis functions; we wish to choose a best partial basis consisting of n $(<N)$ of them to approximate a given function f. The problem is then

$$(7) \qquad \underset{\substack{a_{i_1}, \cdots, a_{i_n} \\ 1 \leq i_1 < \cdots < i_n \leq N}}{\text{minimize}} \quad \| f - \sum_{j=1}^{n} a_{i_j} h_{i_j} \| \ .$$

Assuming we can compute the error

$$\min_{a_{i_j}} \quad \| f - \sum_{j=1}^{n} a_{i_j} h_{i_j} \|$$

for each fixed set of basis functions $h_{i_1}, \cdots, h_{i_n}$, problem (7) can be solved in a brute force way by computing the error corresponding to each of the $\binom{N}{n}$ partial bases. However the computer time required might well be prohibitive; hence a more systematic method of inspecting the partial bases is desirable.

The algorithm we present is a direct generalization of an algorithm due to Hocking and Leslie [5] for selecting a best subset of independent variables in a linear regression analysis; their concern was a least squares fit. The algorithm searches for $r = N - n$ basis functions to delete from the full set of basis functions. We first compute for $i = 1, \cdots, N$,

$$E_i = \min_{a_1, \cdots, a_{i-1}, a_{i+1}, \cdots, a_N} \| f - \sum_{\substack{j=1 \\ j \neq i}}^{N} a_j h_j \|$$

= error of a best approximation using all basis functions except h_i .

To avoid cumbersome notation we assume the basis functions are renumbered so that $E_1 \leq E_2 \leq \cdots \leq E_N$. The following simple lemma is the principle upon which the algorithm is based.

LEMMA 1. Let $I = \{i_1, \cdots, i_r\} \subset \{1, \cdots, N\}$ be a set of r indices with largest element i_r . If

$$(8) \qquad \min_{\{a_i \,:\, i \notin I\}} \| f - \sum_{\substack{i=1 \\ i \notin I}}^{n} a_i h_i \| \leq E_{i_r + 1} ,$$

then for any subset $I' \subset \{1, \cdots, N\}$ which contains an index $i' \geq i_r + 1$ we have

$$\min_{\{a_i \,:\, i \notin I'\}} \| f - \sum_{\substack{i=1 \\ i \notin I'}}^{n} a_i h_i \| \geq \min_{\{a_i \,:\, i \notin I\}} \| f - \sum_{\substack{i=1 \\ i \notin I}}^{n} a_i h_i \| .$$

In words, if inequality (8) is satisfied, a better partial basis cannot be obtained by deleting a basis function with index $\geq i_r + 1$.

Proof. $$\min_{\{a_i : i \notin I'\}} \|f - \sum_{\substack{i=1 \\ i \notin I'}}^{N} a_i h_i\| \geq \min_{\{a_i : i \neq i'\}} \|f - \sum_{\substack{i=1 \\ i \neq i'}}^{N} a_i h_i\|$$

$$= E_{i'} \geq E_{i_r + 1} \geq \min_{\{a_i : i \notin I\}} \|f - \sum_{\substack{i=1 \\ i \notin I}}^{N} a_i h_i\| .$$

We now formulate the algorithm.

Step 1. Compute $E_1, \cdots, E_N$, renumbering the basis functions if necessary. If

$$\min_{a_{r+1}, \cdots, a_N} \|f - \sum_{i=r+1}^{N} a_i h_i\| \leq E_{r+1}$$

then by Lemma 1 with $I = \{1, \cdots, r\}$ we see that $h_{r+1}, \cdots, h_N$ is a best partial basis and the algorithm terminates. If

$$\min_{a_{r+1}, \cdots, a_N} \|f - \sum_{i=r+1}^{N} a_i h_i\| > E_{r+1}$$

we proceed to Step 2.

Step 2. Compute the quantities

$$\min_{\{a_i : i \notin I_2\}} \|f - \sum_{\substack{i=1 \\ i \notin I_2}}^{N} a_i h_i\|$$

where I_2 consists of r of the indices $1, \cdots, r+1$.

Let I_2^* be an index set giving the minimum over such index sets.

If

$$\min_{\{a_i : i \notin I_2^*\}} \|f - \sum_{\substack{i=1 \\ i \notin I_2^*}}^{N} a_i h_i\| \leq E_{r+2}$$

then by Lemma 1 with $I = I_2^*$ a best partial basis is obtained by deleting those basis functions with indices in I_2^* and the algorithm terminates. If

$$\min_{\{a_i : i \notin I_2^*\}} \left\| f - \sum_{\substack{i=1 \\ i \notin I_2^*}}^{N} a_i h_i \right\| > E_{r+2} \ ,$$

we proceed to the next step.

A general step of the algorithm can be described as follows.

Step q. Compute the quantities

$$\min_{\{a_i : i \notin I_q\}} \left\| f - \sum_{\substack{i=1 \\ i \notin I_q}}^{N} a_i h_i \right\|$$

where I_q consists of exactly r of the indices $1, \cdots, r + q - 1$. Let I_q^* be an index set giving the minimum over such index sets. If

$$\min_{\{a_i : i \notin I_q^*\}} \left\| f - \sum_{\substack{i=1 \\ i \notin I_q^*}}^{N} a_i h_i \right\| \leq E_{r+q} \ ,$$

then by Lemma 1 with $I = I_q^*$ a best partial basis is obtained by deleting those basis functions with indices in I_q^* . If

$$\min_{\{a_i : i \notin I_q^*\}} \left\| f - \sum_{\substack{i=1 \\ i \notin I_q^*}}^{N} a_i h_i \right\| > E_{r+q}$$

the algorithm proceeds to the next step.

Of course the algorithm will either terminate at some step with a best partial basis or continue to search all partial bases;

hence a best partial basis will be obtained. The value of the algorithm is in identifying a best partial basis after having searched only a fraction of the $\binom{N}{n}$ possibilities.

We have written a FORTRAN computer program to implement the algorithm for the antenna design problem. It is presently capable of doing weighted least squares problems; we plan to extend it to solve minimax problems. As an illustration of the procedure consider the following example.

EXAMPLE 1.

$$f(u) = \begin{cases} 1 & \text{if } 0 \leq u \leq .1 \\ 0 & \text{if } .15 \leq u \leq .5 \end{cases}$$

The problem is to choose 9 out of the 21 basis functions $\cos 2\pi ku$, $k = 0,1,\cdots,20$, to best approximate $f(u)$ in the least squares norm

$$||f(u) - p(u)|| = \left\{ \int_{[0,.1] \cup [.15,.5]} |f(u) - p(u)|^2 du \right\}^{1/2} .$$

To avoid square roots, we actually work with $||f - p||^2$. The algorithm identifies a best partial basis consisting of the basis functions corresponding to $k = 0,1,2,3,5,6,7,9,10$; notice this gives a much smaller aperture than the maximum allowable. A best partial basis is actually attained in Step 2, although it cannot be identified as such by the algorithm until after Step 3. The algorithm examines only

$$\binom{r + q - 1}{r} = \binom{14}{12} = 91$$ partial bases out of the possible $\binom{21}{9} = 273{,}930$. It should be noted, however,

that the algorithm works exceptionally well for this example because the basis functions are nearly orthogonal.

EXAMPLE 2. For $f(u)$ as in Example 1, the problem is to choose 4 out of the 21 basis functions $\cos 2\pi k(.1)u$, $k = 0, 1, \cdots, 20$, to minimize

$$\int_{[0,.1] \cup [.15,.5]} [f(u) - \sum_{j=1}^{4} a_{k_j} \cos 2\pi k_j(.1)u]^2 + (.01) \sum_{j=1}^{4} a_{k_j}^2 .$$

This criterion has the desirable effect of controlling the size of the coefficients; also numerical difficulties due to ill-conditioning are reduced. The algorithm identifies $\cos 2\pi(.7)u$, $\cos 2\pi(.8)u$, $\cos 2\pi(1.6)u$, $\cos 2\pi(2)u$ as a best partial basis after Step 4 after examining 1140 out of the possible 5985 partial bases.

Figure 1 shows $p(u)$ corresponding to a best partial basis for Example 1.

In [6] an improvement of the Hocking-Leslie algorithm is given; this modification can be incorporated into our algorithm as well.

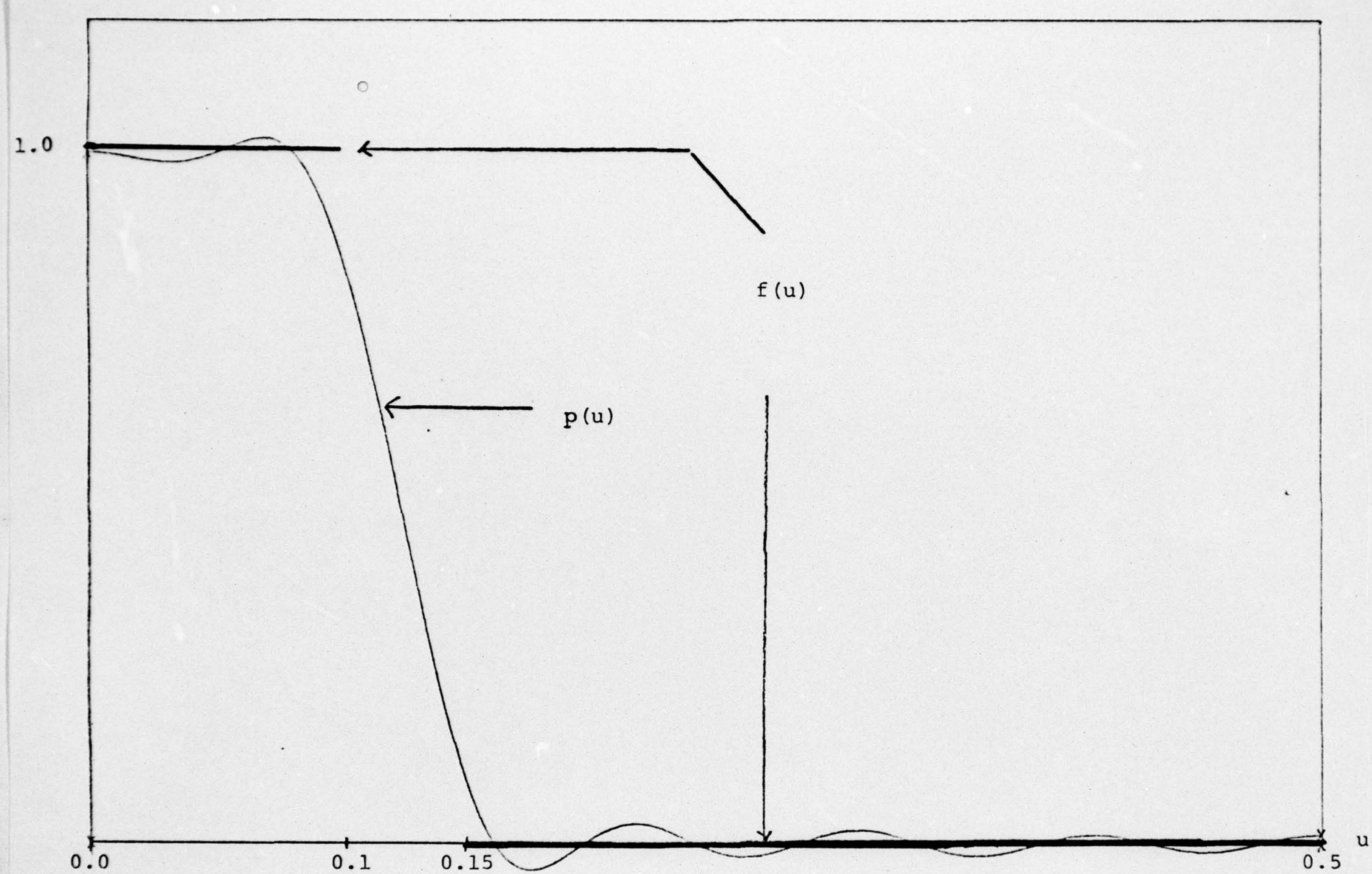

Fig. 1. p(u) is a best partial basis approximation to f(u) in Example 1

3. THEORETICAL RESULTS

In this section we state and prove a "continuous dependence" theorem and an existence theorem and apply them to the antenna design problem. The following lemmas will be used in proving the theorems. We will use the notation

$$\| a \|_1 = \| (a_1, \cdots, a_n) \|_1 = \sum_{i=1}^{n} |a_i|$$

for the ℓ_1-norm of an n-vector of real numbers.

LEMMA 2. Let X be a real normed linear space with norm $\|\cdot\|$, f in X, $\lim_{k\to\infty} \|a^{(k)} - a^{(0)}\|_1 = 0$,

and $\lim_{k\to\infty} \|h_i^{(k)} - h_i^{(0)}\| = 0$ for $i = 1, \cdots, n$

where $h_i^{(k)}$ in X, $i = 1, \cdots, n$; $k = 0, 1, \cdots$.

Then $\lim_{k\to\infty} \|f - \sum_{i=1}^{n} a_i^{(k)} h_i^{(k)}\| = \|f - \sum_{i=1}^{n} a_i^{(0)} h_i^{(0)}\|$.

Proof. $$\left| \|f - \sum_{i=1}^{n} a_i^{(k)} h_i^{(k)}\| - \|f - \sum_{i=1}^{n} a_i^{(0)} h_i^{(0)}\| \right|$$

$$\leq \|(f - \sum_{i=1}^{n} a_i^{(k)} h_i^{(k)}) - (f - \sum_{i=1}^{n} a_i^{(0)} h_i^{(0)})\|$$

$$= \| \sum_{i=1}^{n} a_i^{(k)} h_i^{(k)} - \sum_{i=1}^{n} a_i^{(0)} h_i^{(0)} \|$$

$$\leq \| \sum_{i=1}^{n} a_i^{(k)} (h_i^{(k)} - h_i^{(0)}) \| + \| \sum_{i=1}^{n} (a_i^{(k)} - a_i^{(0)}) h_i^{(0)} \|$$

$$\leq \| a^{(k)} \|_1 \cdot \max_{1\leq i\leq n} \|h_i^{(k)} - h_i^{(0)}\| + \|a^{(k)} - a^{(0)}\|_1 \cdot \max_{1\leq i\leq n} \|h_i^{(0)}\| .$$

The conclusion of the lemma now follows since the convergent sequence $||a^{(k)}||_1$ is bounded.

LEMMA 3. If $\lim_{k\to\infty} ||h_i^{(k)} - h_i^{(0)}|| = 0$, $i = 1,\cdots,n$, where $h_i^{(k)}$ are in the real normed linear space X for $i = 1,\cdots,n$; $k = 0, 1,\cdots$; $h_1^{(0)},\cdots,h_n^{(0)}$ are linearly independent, and $\lim_{k\to\infty} ||a^{(k)}||_1 = \infty$, then $\{||\sum_{i=1}^n a_i^{(k)}h_i^{(k)}||:k = 1, 2,\cdots\}$ is unbounded.

Proof. Assume $||\sum_{i=1}^n a_i^{(k)}h_i^{(k)}||$ is bounded.

The sequence $a^{(k)}/||a^{(k)}||_1$ is bounded and hence has a convergent subsequence $a^{(k_m)}/||a^{(k_m)}||_1$ with limit, say, $a^{(0)}$ having $||a^{(0)}||_1 = 1$. Then $0 = \lim_{m\to\infty} ||\sum_{i=1}^n a_i^{(k_m)} h_i^{(k_m)}|| / ||a^{(k_m)}||_1$

$$= \lim_{m\to\infty} \left|\left| \sum_{i=1}^n \left(\frac{a_i^{(k_m)}}{||a_i^{(k_m)}||_1} \right) h_i^{(k_m)} \right|\right|$$

$$= \left|\left|\sum_{i=1}^n a_i^{(0)}h_i^{(0)}\right|\right| \quad \text{by lemma 2.}$$

Hence $\sum_{i=1}^n a_i^{(0)}h_i^{(0)} = 0$ which contradicts the linear independence of $h_1^{(0)},\cdots,h_n^{(0)}$ and completes the proof of the lemma.

Our first theorem concerns the continuous dependence of a best approximation on the basis functions.

THEOREM 1. Let X be a real normed linear space, $f \in X$, and $h_1^{(0)},\cdots,h_n^{(0)}$ linearly independent in X.

Let $d^{(0)} = ||f - \sum_{i=1}^{n} a_i^{(0)} h_i^{(0)}|| = \inf_{a_1,\cdots,a_n} ||f - \sum_{i=1}^{n} a_i h_i^{(0)}||$.

Then

(i) If $h_1,\cdots,h_n$ are in X and $||h_i - h_i^{(0)}|| < \delta$, $i = 1,\cdots,n$ then $d \le d^{(0)} + \delta \cdot \sum_{i=1}^{n} |a_i^{(0)}|$ where $d = \inf_{a_1,\cdots,a_n} ||f - \sum_{i=1}^{n} a_i h_i||$.

(ii) Given $\varepsilon > 0$ there exists $\delta > 0$ such that if $h_1,\cdots,h_n$ are in X and $||h_i - h_i^{(0)}|| < \delta, i=1,\cdots,n$, then $d^{(0)} - \varepsilon < d$.

(iii) If $\sum_{i=1}^{n} a_i^{(0)} h_i^{(0)}$ is a <u>unique</u> best approximation to f by linear combinations of $h_1^{(0)},\cdots,h_n^{(0)}$, then given $\varepsilon > 0$ there exists $\delta > 0$ such that if $h_1,\cdots,h_n$ are in X and $||h_i - h_i^{(0)}|| < \delta$, $i = 1,\cdots,n$, we have $|a_i - a_i^{(0)}| < \varepsilon$, $i = 1,\cdots,n$, where $||f - \sum_{i=1}^{n} a_i h_i|| = \inf_{a_1,\cdots,a_n} ||f - \sum_{i=1}^{n} a_i h_i||$.

Proof. (i) $d = \inf_{a_1,\cdots,a_n} ||f - \sum_{i=1}^{n} a_i h_i|| \le ||f - \sum_{i=1}^{n} a_i^{(0)} h_i||$

$$= ||(f - \sum_{i=1}^{n} a_i^{(0)} h_i^{(0)}) + \sum_{i=1}^{n} a_i^{(0)} (h_i^{(0)} - h_i)||$$

$$\le ||f - \sum_{i=1}^{n} a_i^{(0)} h_i^{(0)}|| + \sum_{i=1}^{n} |a_i^{(0)}| \cdot ||h_i^{(0)} - h_i||$$

$$\le d^{(0)} + \delta \cdot \sum_{i=1}^{n} |a_i^{(0)}| .$$

(ii) Assume (ii) is false. Then there exist $\varepsilon > 0$, $h_i^{(k)}$ in X, $k = 1, 2, \cdots$, with

$$\lim_{k\to\infty} ||h_i^{(k)} - h_i^{(0)}|| = 0, \ i = 1, \cdots, n, \text{ and}$$

$a^{(k)}$, $k = 1, 2, \cdots$, such that

$$||f - \sum_{i=1}^{n} a_i^{(k)} h_i^{(k)}|| \leq d^{(0)} - \varepsilon .$$

By lemma 3, $||a^{(k)}||_1$ is bounded and hence $\{a^{(k)}\}$ has a convergent subsequence, say $\{a^{(k_m)}\}$, with limit, say, a^*. Then

$$d^{(0)} - \varepsilon \geq \lim_{m\to\infty} ||f - \sum_{i=1}^{n} a_i^{(k_m)} h_i^{(k_m)}|| = ||f - \sum_{i=1}^{n} a_i^* h_i^{(0)}||$$

$\geq d^{(0)}$, a contradiction.

(iii) If the conclusion of (iii) is false, then there exist $h_i^{(k)}$ with $\lim_{k\to\infty} ||h_i^{(k)} - h_i^{(0)}|| = 0$, $i=1,\cdots,n$, and $a^{(k)}$ with $||f - \sum_{i=1}^{n} a_i^{(k)} h_i^{(k)}||$

$$= \inf_{a_1, \cdots, a_n} ||f - \sum_{i=1}^{n} a_i h_i^{(k)}|| \leq ||f||$$

such that $||a^{(k)} - a^{(0)}||_1 \geq \varepsilon$ for $k = 1, 2, \cdots$,

By lemma 3, $||a^{(k)}||_1$ is bounded and hence $\{a^{(k)}\}$ has a convergent subsequence $\{a^{(k_j)}\}$ with limit, say, $\hat{a}$. Then $||f - \sum_{i=1}^{n} \hat{a}_i h_i^{(0)}|| = \lim_{j\to\infty} ||f - \sum_{i=1}^{n} a_i^{(k_j)} h_i^{(k_j)}||$

$= d^{(0)}$ by (i), (ii). This contradicts the uniqueness of $a^{(0)}$ and completes the proof of the theorem.

We now wish to prove an existence theorem for the generalization of problem (5) in section 1. The following hypotheses will be useful.

H1. X is a real normed linear space with norm $\|\cdot\|$ and $f \in X$.

H2. $h(u, \xi)$ is a real function on $[\alpha_1, \alpha_2] \times [\beta_1, \beta_2]$ such that for each ξ in $[\beta_1, \beta_2]$, $h(\cdot, \xi)$ is in X.

H3. For any $\beta_1 \leq \xi_1 < \cdots < \xi_n \leq \beta_2$ the functions $h(u, \xi_1), \cdots, h(u, \xi_n)$ are linearly independent.

H4. There exists $K > 0$ such that for any ξ, η in $[\beta_1, \beta_2]$

$$\|h(u,\xi) - h(u, \eta)\| \leq K \cdot |\xi - \eta| .$$

H5. Notation: $T_\Delta = \{(\xi_1, \cdots, \xi_n) : \beta_1 \leq \xi_1 < \cdots < \xi_n \leq \beta_2,$

$$\xi_{i+1} - \xi_i \geq \Delta,\ i=1, \cdots, n-1\}$$

for $0 < \Delta \leq \dfrac{(\beta_2 - \beta_1)}{n-1}$ (so that T_Δ is nonempty).

THEOREM 2 (EXISTENCE). Assume H1-H5. Then there exist $\hat{a}_1, \cdots, \hat{a}_n$, and $\hat{\xi} \in T_\Delta$ such that

$$\|f(u) = \sum_{i=1}^{n} \hat{a}_i\, h(u, \hat{\xi}_i)\| = \inf_{\substack{a_1, \cdots, a_n \\ \xi \in T_\Delta}} \|f(u) - \sum_{i=1}^{n} a_i\, h(u, \xi_i)\|.$$

Proof. Let $a^{(k)}$, $\xi^{(k)} \in T_\Delta$ be such that

$$\lim_{k \to \infty} \|f - \sum_{i=1}^{n} a_i^{(k)} h(u, \xi_i^{(k)})\| = \inf_{\substack{a_1, \cdots, a_n \\ \xi \in T_\Delta}} \|f(u) - \sum_{i=1}^{n} a_i h(u, \xi_i)\| .$$

Since T_Δ is compact, there exists a convergent subsequence $\xi^{(k_j)}$ of $\xi^{(k)}$, with limit, say, $\hat{\xi}$ in T_Δ. Using H4 we see lemma 3 applies with $h_i^{(k)} = h(u, \xi_i^{(k)})$ and so $\|a^{(k_j)}\|_1$ is bounded. Letting $a^{(k_{j_m})}$ be a convergent subsequence with limit $\hat{a}$ we obtain

$$\lim_{m\to\infty} ||f - \sum_{i=1}^{n} a_i^{(k_{j_m})} h(u,\xi_i^{(k_{j_m})})|| = ||f - \sum_{i=1}^{n} \hat{a}_i h(u,\hat{\xi}_i)||$$

by lemma 2, which completes the proof of the theorem.

We now examine the implications of Theorems 1 and 2. Theorem 2 guarantees the existence of best element locations $0 \leq \xi_1^* < \cdots < \xi_n^* \leq L$ and currents $a_1^*, \cdots, a_n^*$ for problem (5); this follows by taking $h(u, \xi_i) = \cos \xi_i u$. Theorem 1, part (i), guarantees that if $0 \leq \xi_1 < \cdots < \xi_n \leq L$ satisfy $|\xi_i - \xi_i^*| < \gamma$, there is a constant C such that

$$d^* \leq d \leq d^* + C\cdot\gamma \sum_{i=1}^{n} |a_i^*| \tag{9}$$

where $d = \inf_{a_1,\cdots,a_n} ||f - \sum_{i=1}^{n} a_i \cos \xi_i u||$,

$d^* = ||f - \sum_{i=1}^{n} a_i^* \cos \xi_i^* u||$, and $||\cdot||$

is of the form (3) or (4) in section 1.

We can interpret inequality (9) as giving a bound on the deviation of a best approximation to f using element locations which are not optimal (not a solution of problem (5)); suboptimal locations could result from the discretization of possible locations introduced at the end of section 1. Inequality (9) can also be interpreted as bounding the change in the deviation due to a perturbation in the element locations, which might occur physically.

In closing, we mention the related work of Rice ([7], Chapter 8); there the situation is more complicated because $\xi \in T_\Delta$ is not assumed.

Other work on the partial basis problem can be found in [8-11].

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The following problem comes from that of choosing best locations for antenna elements in a linear array. Given a function f and linearly independent functions $h_1, h_2, \cdots, h_N$, choose, for a given $n(<N)$, n of the h's to give a closest linear approximation to f. Both a computational algorithm and theoretical results are given.

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